Generalized oscillatory space in time scales and applications

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Outline

Plan

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Stability analysis of inertial neural networks: A case of almost anti-periodic environment

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Abstract

In this paper, a class of inertial neural networks with time delays is considered. By developing an approach based on differential inequality techniques coupled with Lyapunov function method, some assertions are demonstrated to guarantee the

* Arbi, A., Tahri, N., Jammazi, C., Huang, C. and Cao, J., 2022. Almost anti-periodic solution of inertial neural networks with leakage and time-varying delays on timescales. Circuits, Systems, and Signal Processing, 41(4), pp.1940-1956.

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Almost Anti-periodic Solution of Inertial Neural Networks with Leakage and Time-Varying Delays on Timescales

Adnène Arbi[1,2] · Najeh Tahri[1] · Chaker Jammazi[1,3] · Chuangxia Huang[4,5] · Jinde Cao[6]

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Abstract
This paper studies a class of inertial neural networks with leakages and varying delays on timescales:

$$x_i^{\Delta\Delta}(t) = -a_i(t)x_i^{\Delta}(t-\eta_i(t)) - b_i(t)x_i(t-\xi_i(t)) + \sum_{j=1}^{n} c_{ij}(t) f_j(x_j(t)) + \sum_{j=1}^{n} d_{ij}(t) g_j(x_j(t-q_{ij}(t))) + S_i(t).$$

The problems of the existence, the uniqueness and the exponential stability of almost anti-periodic solution on timescales are investigated. We establish some sufficient conditions to guarantee the main results, by constructing the Lyapunov functions and using some classical inequalities. A numerical example is given for illustration.

* Arbi, A. and Tahri, N., 2022. New results on time scales of pseudo Weyl almost periodic solution of delayed QVSICNNs. Computational and Applied Mathematics, 41(6), pp.1-22.

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New results on time scales of pseudo Weyl almost periodic solution of delayed QVSICNNs

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Abstract
In this work, the problems of investigation of pseudo Weyl almost periodic solution of quaternion-valued shunting inhibitory model of type cellular neural networks with time-varying delays on time space scales are considered. We first introduce the notion of pseudo Weyl almost periodicity on time scales. Next, using fixed point theorem, the theory of time scales, Hölder's inequality and Gronwall's inequality, we give sufficient conditions for the existence and the stability. Finally, we present a numerical application to illustrate the feasibility of our outcomes.

Keywords Quaternion-valued shunting inhibitory cellular neural networks · Pseudo Weyl almost periodic · Time space scales

Mathematics Subject Classification 46S05 · 68T07 · 39A24 · 34N05

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doi:10.3934/dcdss.2022136

DYNAMICS OF DELAYED CELLULAR NEURAL NETWORKS IN THE STEPANOV PSEUDO ALMOST AUTOMORPHIC SPACE

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Abstract. Pseudo almost automorphy (PAA) is a natural generalization of Bochner almost automorphy and Stepanov almost automorphy. Therefore, the results of the existence of PAA solutions of differential equations are few, and the results of the existence of pseudo almost automorphic solutions of difference equations are rare. In this work, we are concerned with a model of delayed cellular neural networks (CNNs). The delays are considered in varying-time form. By the Banach's fixed point theorem, Stepanov like PAA, and constructing a novel Lyapunov functional, we fixed a sufficient criteria that agreement the existence and the Stepanov-exponential stability of Stepanov-like PAA solution of this model of CNNs are obtained. In addition, a numerical example and simulations are performed to verify our theoretical results.

- Harald Bohr : 1924-1926 (almost periodic)
  - Relative density : $\forall \varepsilon > 0$, $\exists L > 0$ such that for any $a \in \mathbb{R}$, $\exists \tau \in [a, a + L]$ verified $\|\Phi(. + \tau) - \Phi(.)\|_{\mathbb{Y}} \leq \varepsilon$.
  - Sequential characterization : for all $(\Phi(. + x_n))_n$, extract a convergent subsequence $\left(\Phi(. + x_{\varphi(n)})\right)_n$ for the norm $\|.\|_{\mathbb{Y}}$.
  - Approximation characterization : there is a trigonometric polynomial sequence $(P_n)_n$ which converges to Φ for the norm $\|.\|_{\mathbb{Y}}$, où $P_n(x) = \sum_{k=1}^{N_n} a_{k,n} e^{ix\lambda_{k,n}}$.
- Vyacheslav Stepanov : 1926 (Stepanov almost periodic)
- Hermann Weyl : 1927 (Weyl almost periodic)
- Bochner : 1962-1964 (almost automorphic)
- Zhang : 1994-1996 (Pseudo almost periodic)

Plan

What is the Time Scales ?

A **time scale** is any closed subset of the real line $\mathbb{R}$ denoted by $\mathbb{T}$, $(\mathbb{R},\ \mathbb{Z},\ \mathbb{N},\ [-1,1]\cup\mathbb{Z},\ q^{\mathbb{Z}})$.

- The forward jump operator :$\sigma(t) := \inf\{s \in \mathbb{T} : s > t\}$.
- The backward jump operator : $\rho(t) := \sup\{s \in \mathbb{T} : s < t\}$.
- The graininess function : $\mu(t) := \sigma(t) - t$.

t right-dense	$\sigma(t) = t$	t right-scattered	$\sigma(t) > t$
t left-dense	$\rho(t) = t$	t left-scattered	$\rho(t) < t$
t dense	$\rho(t) = t = \sigma(t)$	t isolated	$\rho(t) < t < \sigma(t)$

Hilger S. (1988). *Ein Maβkettenkalkul mit Anwendung auf Zentrumsmanningfaltigkeiten*. PhD Thesis, Universitat Wurzburg.

Bohner M. & Peterson A. (2001). *Dynamic Equations on Time Scales : An introduction with applications*. Birkhäuser : Basel.

$\triangle$-derivative

Let $f:\mathbb{T}\to\mathbb{R}$ be a function, we say that f admits a $\triangle$-derivative $\left(f^{\triangle}\right)$ if there exists $\varepsilon>0$ such that

$$|[f(\sigma(t))-f(s)]-f^{\triangle}(t)[\sigma(t)-s]|\leq\varepsilon|\sigma(t)-s|,\quad\forall t\in\mathbb{T}^{\kappa}. \qquad (1)$$

We say that $g(t)$ is antiderivate of $f(t)$ if $g^{\triangle}(t)=f(t)$, for all $t\in\mathbb{T}^{\kappa}$.

Theorem

Suppose that $f,g:\mathbb{T}\to\mathbb{R}$ two functions $\triangle$-differentiable in $t\in\mathbb{T}^{\kappa}$. Then

- $(f+\lambda g)^{\triangle}(t)=f^{\triangle}(t)+\lambda g^{\triangle}(t)$, *for any constant* λ.
- $(f.g)^{\triangle}(t)=f^{\triangle}(t)g(t)+f^{\sigma}(t)g^{\triangle}(t)=f(t)g^{\triangle}(t)+f^{\triangle}(t)g^{\sigma}(t)$.
- *If* $f(t)f^{\sigma}(t)\neq 0$, *then* $\left(\frac{1}{g}\right)^{\triangle}=-\frac{f^{\triangle}(t)}{f(t)f^{\sigma}(t)}$
- *If* $g(t)g^{\sigma}(t)\neq 0$ *then* $\left(\frac{f}{g}\right)^{\triangle}=\frac{f^{\triangle}(t)g(t)-f(t)g^{\triangle}(t)}{g(t)g^{\sigma}(t)}$.

rd-continuous functions and regressive group

- The function $f : \mathbb{T} \longrightarrow \mathbb{R}$ is called rd-continuous if it is continuous at all right-dense points of $\mathbb{T}$ and if left-hand limits exist as a finite number at left-dense points of $\mathbb{T}$. The set of all rd-continuous functions $f : \mathbb{T} \longrightarrow \mathbb{R}$ is denoted by $C_{rd}(\mathbb{T}, \mathbb{R})$.
- $\mathcal{R}(\mathbb{T}, \mathbb{R}) = \{p : \mathbb{T} \longrightarrow \mathbb{R} \text{ rd-continuous} : \ 1 + \mu(t)p(t) \neq 0\}$.
- $(\mathcal{R}, \oplus)$ is an Abelian group under the $\oplus$ and $\ominus$ defined by

$$p \oplus q := p + q + \mu pq, \quad p \ominus q := \frac{p - q}{1 + \mu q}. \tag{2}$$

Bohner M. & Peterson A. (2003). *Advances in Dynamic Equations on Time Scales*. Boston, MA : Birkhäuser.

Anatoly A. Martynyuk (2016). *Stability Theory for Dynamic Equations on Time Scales*. Birkhäuser, Switzerland.

The exponential function

Let $p \in \mathcal{R}(\mathbb{T},\mathbb{R})$ and $s \in \mathbb{T}$, the exponential function on time scales defined by

$$e_p(t,s) := \begin{cases} \exp\left(\int_s^t p(\tau)\,\mathrm{d}\tau\right), & t \in \mathbb{T}, \quad \mu = 0 \\ \exp\left(\int_s^t \frac{\log(1+\mu(\tau)p(\tau))}{\mu(\tau)} \triangle\tau\right), & t \in \mathbb{T}, \quad \mu > 0. \end{cases}$$

Let $p, q \in \mathcal{R}(\mathbb{T},\mathbb{R})$ and $a, b, c \in \mathbb{T}$, then

- $e_0(t,s) = e_p(t,t) = 1$,
- $e_p(\sigma(t),s) = (1+\mu(t)p(t))e_p(t,s)$,
- $e_p(t,s) = e_{\ominus p}(s,t) = \frac{1}{e_p(s,t)}$,
- $e_p(t,s)e_q(t,s) = e_{p\oplus q}(t,s)$,
- $e_p(t,r)e_p(r,s) = e_p(t,s)$,
- $\frac{e_p(t,s)}{e_q(t,s)} = e_{p\ominus q}(t,s)$,
- $[e_p(c,.)]^{\triangle} = -p(.)[e_p(c,.)]^{\sigma}$,
- $\int_a^b p(t)e_p(c,\sigma(t))\triangle t = e_p(c,a) - e_p(c,b)$.

Plan

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Time scale Π is invariant under translations

A time scale Π is called invariant under translations if

$$\Pi = \{\theta \in \mathbb{R} : \ \theta \pm t \in \mathbb{T}, \ \forall t \in \mathbb{T}\} \neq \{0\}.$$

In the rest of the sequence, Π is assumed to be invariant under translations.

Stepanov almost automorphic functions

Definition

A function $f : \mathbb{R} \times \mathbb{R} \to \mathbb{R}$ such that $f(., u) \in BS^p(\mathbb{R}, \mathbb{R})$ for each $u \in \mathbb{R}$ is said to be Stepanov-like pseudo almost automorphic if f is written in the following form :

$$f = g + \phi,$$

where $g \in S^p AP(\mathbb{R} \times \mathbb{R}, \mathbb{R})$ and $\phi \in S^p PAA_0(\mathbb{R} \times \mathbb{R}, \mathbb{R})$. The space of all such functions will be denoted by $S^p PAA(\mathbb{R} \times \mathbb{R}, \mathbb{R})$.

Lemma

1) If $h, g \in S^p PAA(\mathbb{R}, \mathbb{R})$, then $h + g, hg \in S^p PAA(\mathbb{R}, \mathbb{R})$.
2) If $h \in S^p PAA(\mathbb{R}, \mathbb{R})$ and $g \in S^p AP(\mathbb{R}, \mathbb{R})$, then $hg \in S^p PAA(\mathbb{R}, \mathbb{R})$.

Lemma

$(S^p PAA(\mathbb{R}, \mathbb{R}), \| \, . \, \|_{S^p})$ is a Banach space, where

$$\|g\|_{S_l^p} := \lim_{l \to +\infty} \sup_{\alpha \in \mathbb{T}} \left\{ \frac{1}{l} \int_{\alpha}^{\alpha + l} \|g\|_{\mathbb{E}}^p \triangle t \right\}^{\frac{1}{p}},$$

$$\|g\|_{W^p} := \lim_{l \to +\infty} \|g\|_{S_l^p}.$$

Weyl almost automorphic functions

Definition

Let $f \in L^p_{loc}(\mathbb{T}, \mathbb{A})$. f is said p-th Weyl almost automorphic if for any sequence $(s_n)_{n\in\mathbb{N}} \subset \Pi$, there exists a subsequence $(s_{n_k})_{k\in\mathbb{N}}$ and function $\bar{g} \in L^p_{loc}(\mathbb{T}, \mathbb{E})$ verified

$$\|f(t+s_{n_k})-\bar{g}(t)\|_{W^p} \longrightarrow 0 \ , \|\bar{g}(t-s_{n_k})-f(t)\|_{W^p} \longrightarrow 0, \quad \text{as } k \longrightarrow +\infty.$$

Denote by $AAW^p(\mathbb{T}, \mathbb{A})$ the set of all such functions.

Remarque

$AA(\mathbb{T}, \mathbb{E}) \subset AAW^p(\mathbb{T}, \mathbb{E})$

Pseudo Weyl almost automophic functions

Definition

A function $h \in \mathcal{BC}_{rd}(\mathbb{T}, \mathbb{A})$ is said Weyl-ergodic if

$$\lim_{r \to +\infty} \frac{1}{r} \int_{-r}^{r} \left\{ \lim_{l \to +\infty} \frac{1}{l} \int_{s-l}^{s+l} \|h(t)\|^p \triangle \mathrm{t} \right\}^{\frac{1}{p}} \triangle \mathrm{s} = 0. \quad (3)$$

The set of all functions denoted by $\mathcal{E}(\mathbb{T}, \mathbb{A})$.

Definition

Let $f \in L^p_{loc}(\mathbb{T}, \mathbb{A})$. f is called pseudo p-th Weyl almost automophic if it can be represented as

$$f = g + h, \quad (4)$$

where $g \in AAW^p(\mathbb{T}, \mathbb{A})$ and $h \in \mathcal{E}(\mathbb{T}, \mathbb{A})$. Denote the set of all such functions by $PAAW^p(\mathbb{T}, \mathbb{A})$.

Remarque

Unlike the almost automophic concept and their extensions like pseudo almost automophic and weighted pseudo almost automophic, the unique decomposition of the pseudo weyl almost automophic in equation (4) *functions is not guaranteed.*

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Cellular neural networks model

$$x_i'(t) = -a_i(t)x_i(t) + \sum_{m=1}^{n} b_{ij}(t)f_j(x_j(t)) + \sum_{m=1}^{n} c_{ij}(t)g_j(x_j(t-\xi_j(t))) + I_i(t), \tag{5}$$

Where $i \in \{1, 2, ..., n\}$, n correspon ds to the number of units in neural networks, $x_i(t) \in \mathbb{R}$ correspon ds to the state of the *ith* unit at time t, $a_i(t) = diag(a_1(t), a_2(t), ..., a_n(t))$ represents the rate with which the ith neuron will reset its potential to the resting state in isolation when they are disconnected from the network and the external inputs at time t, f_j, $g_j : \mathbb{R} \to \mathbb{R}$ are output transfer functions, $b_{ij}(.)$ and $c_{ij}(.)$ present the connection weights and the discretely delayed connection weights of the *jth* neuron on the i neuron, respectively. $\xi_j(.)$ correspon ds to transmission delays at time t and satisfy $t - \xi_j(t) \in \mathbb{R}$ for $t \in \mathbb{R}$, $I_i(.)$ denote the state bias of the *ith* neuron. The initial condition of system (5) is of the form

$$x_i(s) = v_i(s), \quad s \in (-\infty, 0],$$

Hypotheses

(H_1) : The functions $a_{ij}(.)$, $b_{ij}(.)$, $c_{ij}(.)$, $d_{ij}(.) \in S^p PAA(\mathbb{R}, \mathbb{R})$ and $\xi_j(.) \in S^p AA(\mathbb{R}, \mathbb{R}) \cap C^1(\mathbb{R}, \mathbb{R})$ such that

$$0 \leq \xi_j(.) \leq \overline{\xi}, \quad 0 \leq \xi^* - \xi_j'(.) < 1 - \xi_j'(.).$$

(H_2) : There exist positive constants L_i^f, L_i^g, L_{h_i} such that for any u, $v \in \mathbb{R}$, the activity functions f_i, g_i, $h_i \in C(\mathbb{R}, \mathbb{R})$ satisfy

$$\begin{aligned} \mid f_i(u) - f_i(v) \mid &\leq L_i^f \mid u - v \mid, \\ \mid g_i(u) - g_i(v) \mid &\leq L_i^g \mid u - v \mid . \end{aligned}$$

Furthermore, we suppose that $f_i(0) = g_i(0) = 0$.

(H_3) : $M < 1$.

Lemma

Let $\psi = (\psi_1, ..., \psi_n) \in S^p PAA(\mathbb{R}, \mathbb{R})$. *Under assumptions* (H_1) *and* (H_2), *the nonlinear operator defined by,*

$$(\mathbb{P}_\psi)_i(t) = \int_{-\infty}^{t} e^{-\int_s^t a_i(u)du} K_i(s)ds, \quad i = 1, ..., n.$$

Where
$K_i(t) = \sum_{m=1}^{n} b_{ij}(t) f_j(\psi_j(t)) + \sum_{m=1}^{n} c_{ij}(t) g_j(\psi_j(t - \xi_j(t))) + I_i(t)$,
maps $S^p PAA(\mathbb{R}, \mathbb{R})$ *into itself.*

Theorem

Assume that the conditions $(H_1 - H_3)$ are satisfied. Then, system (5) has a unique S^p-pseudo almost automorphic solution in the region

$$\mathbb{B} = \{\psi : \psi \in S^p PAA(\mathbb{R}, \mathbb{R}), \| \psi - \psi_0 \|_{S^p} \leq r^*\}.$$

Theorem

Suppose that assumptions $(H_1 - H_3)$ hold. Then the unique Stepanov-like pseudo almost automorphic solution of system (5) is S^p-exponentially stable whenever

$$(H_4) \ : \ \frac{1}{a_i^*} - \check{a}_i < - \sum_{m=1}^{n} a_i^* \left(b_{ij}^* L_i^f\right)^2 - \sum_{m=1}^{n} \frac{a_i^* \left(c_{ij}^* L_i^g\right)^2}{1-\xi^*}.$$

Proof

$$V(t) = \sum_{i=1}^{n} \mid X_i(t) \mid^2 \hat{e}^{\lambda t} + \frac{\exp(\lambda(\bar{\mu}+\bar{\xi}))}{1-\xi^*} \sum_{i=1}^{n} \sum_{m=1}^{n} a_i^* \left(c_{ij}^* L_i^g\right)^2 \int_{t-\xi(t)}^{t} \mid X_i(z) \mid^2 \hat{e}^{\lambda z} dz.$$

where $X(.) = u(.) - v(.)$.

Clifford neural networks model

1. As a generalization of real-valued neural networks, complex-valued neural networks and quaternion-valued neural networks, Clifford-valued neural networks have been proved to have more advantages than real-valued neural networks in dealing with high-dimensional data and spatial transformation
2. The importance of quaternions is justified by their various applications in quantum physics where they are used to translate rotations by taking into account the spin of a particle. Quaternions are also used in computer graphics, for example they were used to model rotations in the 3D video game Tomb Raider.

$$\begin{cases} z_{ij}^{\triangle}(t) & = -d_{ij}(t)z_{ij}(t) - \sum\limits_{c_{kl}\in N_r(i,j)} b_{ij}^{kl}(t)f_{ij}(z_{kl}(t))z_{ij}(t) \\ & \quad - \sum\limits_{c_{kl}\in N_s(i,j)} c_{ij}^{kl}(t)g_{ij}(z_{kl}(t-\beta_{kl}(t)))z_{ij}(t) + D_{ij}(t), \end{cases} \tag{6}$$

- $ij \in \Gamma := \{11, ..., 1n, ..., m1, ..., mn\}$,
- $d_{ij},\ b_{ij}^{kl},\ c_{ij},\ z_{ij},\ D_{ij} \in \mathbb{A}$,
- $\beta_{kl}(t) \in \mathbb{T}^+$ satisfy $t - \beta_{kl}(t) \in \mathbb{T},\ \forall t \in \mathbb{T}$,
- $d_{ij}(t) = \sum\limits_{B} d_{ij}^{B} e_B \in \mathbb{A}$,
  $d_{ij}^{*}(t) = \sum\limits_{B\neq\emptyset} d_{ij}^{B} e_B \in \mathbb{A}$,
  $d_{ij}^{R}(t) = d_{ij}(t) - d_{ij}^{*}(t)$,
- $d^- = \min\limits_{ij\in\Gamma}\{\inf\limits_{t\in\mathbb{T}}\{d_{ij}^R(t)\}\}$,
- $d^+ = \max\limits_{ij\in\Gamma}\{\sup\limits_{t\in\mathbb{T}}\{d_{ij}^R(t)\}\}$,
- $\tilde{d}^+ = \sup\limits_{t\in\mathbb{T}}\{\|d_{ij}^R(t)\|_{\mathbb{A}}\}$,
- $b_{ij}^{kl^+} = \sup\limits_{t\in\mathbb{T}}\{b_{ij}^{kl}(t)\}$,
- $c_{ij}^{kl^+} = \sup\limits_{t\in\mathbb{T}}\{c_{ij}^{kl}(t)\}$,
- $\beta_{kl}^{+} = \sup\limits_{t\in\mathbb{T}}\{\beta_{kl}(t)\}$,
- $\beta'_{kl} = \max\limits_{ij\in\Gamma}\{\sup\limits_{t\in\mathbb{T}}\{\beta_{kl}^{\triangle}(t)\}\}$.

$$N_r(i,j) := \{c_{pq} :\ max(|p-i|, |q-j|) \leq r,\ 1 \leq p \leq m, 1 \leq q \leq n\}.$$

Assumptions

$(\mathcal{P}_1)$ For $ij \in \Gamma$, there exist $L_{ij}^f > 0$ and $L_{ij}^g > 0$ such that

$$\|f_{ij}(z) - f_{ij}(z^*)\|_{\mathbb{A}} \leq L_{ij}^f \|z - z^*\|_{\mathbb{A}}, \quad \|g_{ij}(z) - g_{ij}(z^*)\|_{\mathbb{A}} \leq L_{ij}^g \|z - z^*\|_{\mathbb{A}}$$

where $f_{ij}(0) = g_{ij}(0) = 0$ for all $z, z^* \in \mathbb{A}$.

$(\mathcal{P}_2)$ For $ij \in \Gamma$, $a_{ij}^R \in AA(\mathbb{T}, \mathbb{R}^+)$ satisfy $-a_{ij}^R \in \mathcal{R}^+$, $\tilde{a}_{ij}^{R^+} \in AA(\mathbb{T}, \mathbb{A})$, $b_{ij}^{kl}, c_{ij}^{kl} \in AA(\mathbb{T}, \mathbb{R}^+)$, $D_{ij} \in PAAW^P(\mathbb{T}, \mathbb{A})$, $\beta_{kl} \in AA(\mathbb{T}, \mathbb{R}^+) \cap C_{rd}^1(\mathbb{T}, \mathbb{T})$, $\beta' < 1$.

Denote $\varphi^0 = (\varphi_{11}^0, \dots, \varphi_{1n}^0, \dots, \varphi_{m1}^0, \dots, \varphi_{mn}^0)$, where

$$\varphi_{ij}^0(t) = \int_{-\infty}^{t} e_{-d^-}(t, \sigma(s)) D_{ij}(s) \triangle s, \quad ij \in \Gamma.$$

Choose $\alpha \geq \|\varphi_{ij}^0(t)\|_{\mathbb{A}}$

$$\Upsilon = \left\{ \psi \in BCU_{rd}(\mathbb{T}, \mathbb{A}^{mn}) : \quad \left\| \psi - \varphi^0 \right\|_{\infty} \leq 2\alpha \right\}.$$

So for each $\psi \in \Upsilon$, we have $\|\psi\|_\infty \leq \left\|\psi - \varphi^0\right\|_\infty + \left\|\varphi^0\right\|_\infty \leq 2\alpha$.

Assumptions

$(\mathcal{P}_3)$

$$K = \frac{2}{d^-} \max_{ij\in\Gamma} \left\{ \tilde{d}_{ij}^+ + \sum_{c_{kl}\in N_r(i,j)} 2\alpha b_{ij}^{kl^+} L_{ij}^f + \sum_{c_{kl}\in N_s(i,j)} 2\alpha c_{ij}^{kl^+} L_{ij}^g \right\} < 1.$$

$(\mathcal{P}_4)$ For $p > 2$

$$\max_{ij\in\Gamma} \left\{ 24 \left(\frac{2p-4}{d^- p}\right)^{p-2} \left(\frac{4}{d^- p}\right) \left[2\left(\tilde{d}_{ij}^+\right)^p + 2\left(\sum_{c_{kl}\in N_r(i,j)} 2\alpha b_{ij}^{kl^+} L_{ij}^f \right)^p + \left(1 + \frac{2e^{\frac{p}{4}} a^+ \beta^+}{1-\beta'}\right) \left(\sum_{c_{kl}\in N_s(i,j)} 2\alpha c_{ij}^{kl^+} L_{ij}^g \right)^p \right] \right\} < 1,$$

and, for $p = 2$

$$\max_{ij\in\Gamma}\left\{24\left(\frac{2}{d^-}\right)^2\left[2\left(\tilde{d}_{ij}^+\right)^2+2\left(\sum_{c_{kl}\in N_r(i,j)}2\alpha b_{ij}^{kl^+}L_{ij}^f\right)^2\right.\right.$$
$$\left.\left.+\left(1+\frac{2e^{\frac{1}{2}}d^+\beta^+}{1-\beta'}\right)\left(\sum_{c_{kl}\in N_s(i,j)}2\alpha c_{ij}^{kl^+}L_{ij}^g\right)^2\right]\right\}<1,$$

Control system of Clifford model

- The synchronisation of two dynamical neural networks is one more important mathematical problem due to its interesting place in the real world application.
- The Clifford algebra is an algebraic structure that generalizes the notion of complex number and quaternion.
- The study of Clifford's algebras is closely related to the theory of quadratic form, and it has important applications in geometry and theoretical physics.
- Their name is derived from that of the mathematician William Kingdon Clifford that introduced since 1878.

The clifford algebra on $\mathbb{R}^n$ is defined as

$$\mathbb{A} = \left\{ \sum_{B \subset \{1,2,\ldots,\chi\}} b^B e_B : \ b^B \in \mathbb{R} \right\},$$

where $e_B = e_{k_1} e_{k_2} \ldots e_{k_v}$, $B = k_1 k_2 \ldots k_v$, $1 \leq k_1 < k_2 < \ldots < k_v \leq \chi$. Besides, $e_\emptyset = e_0 = 1$ and $e_{k_1}, e_{k_2}, \ldots, e_{k_v}$ are labeled Clifford-generators and verify $e_{ij}^2 + 1 = 0$, and $e_{ij} e_j + e_j e_{ij} = 0$, $i \neq j$ for all $i, j = 1, \ldots, \chi$. We always assume that the product of Clifford algebra afterwards without any further comments : $e_{k_1} e_{k_2} \ldots e_{k_v} = e_{k_1 k_2 \ldots k_v}$,
$\Theta = \{\emptyset, \ 1, \ 2, \ \ldots, \ B, \ \ldots \chi\}$, then

$$\mathbb{A} = \left\{ \sum_{B \in \Theta} b^B e_B : \ B^B \in \mathbb{R} \right\}.$$

For $x \in \sum_B x^B e_B \in \mathbb{A}$, and $x =^t (x_1, \ldots, x_n) \in \mathbb{A}^n$, we define

$$\|x\|_{\mathbb{A}} = \max_{B \in \Theta} \left\{ |x^B| \right\}, \quad \|x\|_{\mathbb{A}^n} = \max_{1 \leq p \leq n} \left\{ \|x_p\|_{\mathbb{A}} \right\}.$$

Consider a clifford-valued network control system

$$\left\{ \begin{array}{ll} y_{ij}^{\triangle}(t) & = -d_{ij}(t)y_{ij}(t) - \sum_{c_{kl}\in N_r(i,j)} b_{ij}^{kl}(t)f_{ij}(y_{kl}(t))y_{ij}(t) \\ & \quad - \sum_{c_{kl}\in N_s(i,j)} c_{ij}^{kl}(t)g_{ij}(y_{kl}(t-\beta_{kl}(t)))y_{ij}(t) + D_{ij}(t) + S_{ij}(t), \end{array} \right. \tag{7}$$

where the state is $y_{ij} \in \mathbb{A}$, the control is $S_{ij} \in \mathbb{A}$ and $ij \in \Gamma$.
The synchronization errors between system (6) and system (7) are given by

$$x_{ij}(t) = z_{ij}(t) - y_{ij}(t), \quad ij \in \Gamma.$$

Error system

$$\begin{cases} x_{ij}^{\triangle}(t) = -d_{ij}(t)x_{ij}(t) + \sum_{c_{kl}\in N_r(i,j)} b_{ij}^{kl}(t)\left[f_{ij}(y_{kl}(t))y_{ij}(t) - f_{ij}(z_{kl}(t))z_{ij}(t)\right] \\ \quad + \sum_{c_{kl}\in N_s(i,j)} c_{ij}^{kl}(t)\left[g_{ij}(y_{kl}(t-\beta_{kl}(t)))y_{ij}(t) - g_{ij}(z_{kl}(t-\beta_{kl}(t)))z_{ij}(t)\right] + S_{ij}(t) \end{cases} \tag{8}$$

Let a feedback

$$\begin{cases} S_{ij}(t) = \sum_{A\in\mathbb{A}} S_{ij}^{A}(t)e_A + \sum_{c_{kl}\in N_r(i,j)} b_{ij}^{kl}(t)f_{ij}(z_{kl}(t))z_{ij}(t) - \sum_{c_{kl}\in N_r(i,j)} b_{ij}^{kl}(t)f_{ij}(z_{kl}(t)) \\ \quad + \sum_{c_{kl}\in N_s(i,j)} c_{ij}^{kl}(t)g_{ij}(y_{kl}(t-\beta_{kl}(t)))y_{ij}(t) - \sum_{c_{kl}\in N_s(i,j)} c_{ij}^{kl}(t)g_{ij}(y_{kl}(t-\beta_{kl}(t))) \\ S_{ij}^{A}(t) = -M_{1ij}x_{ij}^{A}(t) - M_{2ij}\left|z_{ij}^{A}(t)\right|^{\alpha} sgn\left(x_{ij}^{A}(t)\right) \\ \quad -M_{3ij}\left|x_{ij}^{A}(t)\right|^{\gamma} sgn\left(x_{ij}^{A}(t)\right) - M_{4ij}\left|\sum_{ij\in\Gamma} x_{ij}^{A}(t-\beta_{ij}(t))\right| sgn\left(x_{ij}^{A}(t)\right) \end{cases} \tag{9}$$

where $ij \in \Gamma$, $A \in \mathbb{A}$, $0 < \alpha < 1$, $\gamma > 1$ and $M_{1ij}, M_{2ij}, M_{3ij}, M_{4ij}$ are the parameters that will be determined.

Theorem

The network (6) has only one solution in the region Υ, as soon as the properties $(\mathcal{P}_1) - (\mathcal{P}_4)$ are fulfilled. Moreover, the of network (6) is exponentially stable.

Theorem

Assume $(\mathcal{P}_1)$ hold. If in addition, M_{1ij}, M_{2ij}, M_{3ij} and M_{4ij} of the feedback (9) satisfy

$$M_{1ij} \geq -d^{-} + \sum_{c_{kl} \in N_r(i,j)} b_{ij}^{kl^+} L_{ij}^{f} \tag{10}$$

$$M_{2ij} > 0, \quad M_{3ij} > 0 \text{ and } M_{4ij} \geq \sum_{c_{kl} \in N_s(i,j)} c_{ij}^{kl^+} L_{ij}^{g}, \tag{11}$$

for all $ij \in \Gamma$. By means of the feedback (9) can reach synchronization the network (8) at fixed time.

In particular, for $\sum_{c_{kl}\in N_r(i,j)} b_{ij}^{kl^+} L_{ij}^f \geq d^-$, we have the following corollary

Corollary

Assume $(\mathcal{P}_1)$ *hold. If in addition,* M_{1ij}, M_{2ij}, M_{3ij} *and* M_{4ij} *of the feedback* (9) *satisfy*

$$M_{1ij} \geq 0 \tag{12}$$

$$M_{2ij} > 0 \tag{13}$$

$$M_{3ij} > 0 \tag{14}$$

$$M_{4ij} \geq \sum_{c_{kl}\in N_s(i,j)} c_{ij}^{kl^+} L_{ij}^g, \tag{15}$$

for all $ij \in \Gamma$. *By means of the feedback* (9), *the system* (6) *and the system* (7) *can reach synchronization at fixed time.*

Plan

In network (6), let $n = 2$, $r = s = 1$, where $\Theta = \{0,\ 1,\ 2,\ 12\}$ and

$$z_{ij}(t) = z_{ij}^{0}(t)e_0 + z_{ij}^{1}(t)e_1 + z_{ij}^{2}(t)e_2 + z_{ij}^{12}(t)e_{12}$$

$$d_{11}(t) = 32\cos^2(1.71t)e_0 + 0.28\sin(2t)e_1 - 0.4\cos(2.23t)e_2 + 0.001\sin^2(3t)e_{12}$$

$$d_{21}(t) = 39\left|\sin(1.41t)\right| e_0 - 0.5\cos(2t)e_1 - 0.33\cos^2(2.64t)e_2 + 0.17\sin(t)e_{12}$$

$$d_{12}(t) = 33\sin^2(t)e_0 - i0.28\cos(5t)e_1 - 0.25\sin^3(2.23t)e_2 + 1.66\sin^2(3t)e_{12}$$

$$d_{22}(t) = 37\cos^4(1.71t)e_0 + 0.25\sin(7t)e_1 - 0.33|\cos(3.31t)|e_2 + 0.12\sin^2(5t)e_{12},$$

$$f_{11}(z) = f_{12}(z) = 0.4\sin(0.25z^0 + 0.28z^2)e_0 - 0.4\left|1.05z^1 + 0.34z^2\right| e_1 + 0.25\cos(0.2z^0)e_2 + 0.25\sin(0.2z^0)e_{12}$$

$$f_{21}(z) = f_{22}(z) = 0.25 \left|0.47z^1 + 0.24z^{12}\right| e_0 - 0.17\sin(0.2z^0 + 0.28z^{12})e_1 - 0.4 \left|1.05z^1 + 0.34z^2\right| e_2 + 0.47\sin(0.2z^1)e_{12}$$

$$g_{11}(z) = g_{12}(z) = 0.05\sin(6.34z^0)e_0 - 0.04\sin(1.4z^{12})e_1 + 0.07\sin(0.33z^2 + 0.33z^1)e_2 + 0.04\sin(5.64z^0)e_{12}$$

$$g_{21}(z) = g_{22}(z) = 0.05 \left|0.66z^1 + 0.33z^2\right| e_0 - 0.04\sin(1.4z^{12} + z^0)e_1 + 0.04\sin(5.64z^0)e_2 - 0.04\sin(1.4z^{12})e_{12}$$

$$D_{11}(t) = D_{12}(t) = 1.41\cos(t)e_0 + 1.33e^{-|t|}e_1 + 0.47\sin(0.5t)e_2 - 0.2\sin(2t)e_{12}$$

$$D_{21}(t) = D_{22}(t) = 0.47\sin(0.5t)e_0 + 0.47\cos(1.41t)e_1 - 0.2\sin(2t)e_2 + e^{-|t|}e_{12}$$

and $k, l = 1, 2 \in N_1(i, j)$ for any $i = j = 1, 2$,

$$b_{ij}^{11}(t) = b_{ij}^{21}(t) = 0.33|\sin(1.41t)|e_0 + 0.16\cos(4t)e_1 + (0.75\cos(2t) + 1)e_2 + 0.25\cos^2(2.23t)e_{12},$$

$$b_{ij}^{12}(t) = b_{ij}^{22}(t) = 0.3|\cos(t)|e_0 + 0.17\sin(3t)e_1 + 0.25\cos(2t)e_2 + 0.21\sin(2t)e_{12},$$

$$c_{ij}^{11}(t) = c_{ij}^{21}(t) = (\sin(1.73t) + 3)e_0 + |\cos(2.23t)|e_1 + 2\sin^2(2.64t)e_2 + 0.25\cos(2t)e_{12},$$

$$c_{ij}^{12}(t) = c_{ij}^{22}(t) = \cos(1.5t)e_0 + \sin(2t)e_1 + \cos^3(2.64t)e_2 + 0.2\sin(2t)e_{12},$$

$$b_{ij}^{kl} : \begin{pmatrix} b_{ij}^{11} & b_{ij}^{12} & b_{ij}^{13} \\ b_{ij}^{21} & b_{ij}^{22} & b_{ij}^{23} \\ b_{ij}^{31} & b_{ij}^{32} & b_{ij}^{33} \end{pmatrix}$$

$$c_{ij}^{kl} : \begin{pmatrix} c_{ij}^{11} & c_{ij}^{12} & c_{ij}^{13} \\ c_{ij}^{21} & c_{ij}^{22} & c_{ij}^{23} \\ c_{ij}^{31} & c_{ij}^{32} & c_{ij}^{33} \end{pmatrix}$$

$$\beta_{11}(t) = 0.04\cos^8(1.41t), \quad \beta_{12}(t) = 0.06\sin^2(1.41t)$$
$$\beta_{21}(t) = 0.02\cos^2(0.28t), \quad \beta_{22}(t) = 0.03\cos^4(0.24t),$$
$$M_{1ij} = 10, M_{2ij} = 5, M_{3ij} = 6, M_{4ij} = 12.$$

After all calculations we have

$$L^f_{11} = L^f_{12} = 0.4, \quad L^f_{21} = L^f_{22} = 0.47$$
$$L^g_{11} = L^g_{12} = 0.07, \quad L^g_{21} = L^g_{22} = 0.05$$
$$\tilde{d}^+_{11} = 0.4, \quad \tilde{d}^+_{12} = 1.66, \quad \tilde{d}^+_{21} = 0.5, \quad \tilde{d}^+_{22} = 0.33$$
$$\sum_{c_{kl}\in N_1(1,1)} b^{kl^+}_{11} = \sum_{c_{kl}\in N_1(1,2)} b^{kl^+}_{12} = \sum_{c_{kl}\in N_1(2,1)} b^{kl^+}_{21} = \sum_{c_{kl}\in N_1(2,2)} b^{kl^+}_{22} = 4.1$$
$$\sum_{c_{kl}\in N_1(1,1)} c^{kl^+}_{11} = \sum_{c_{kl}\in N_1(1,2)} c^{kl^+}_{12} = \sum_{c_{kl}\in N_1(2,1)} c^{kl^+}_{21} = \sum_{c_{kl}\in N_1(2,2)} c^{kl^+}_{22} = 10$$
$$d^- = 32, \; d^+ = 39, \; \beta = 0.16, \; \beta' = 0.5$$

Take $\alpha = 0.06 \geq \|\Phi^0\|$. For $1 \leq i,j \leq 2$ we have

$$K = \frac{2}{d^-} \max_{ij\in\Gamma} \left\{ \tilde{d}^+_{ij} + \sum_{c_{kl}\in N_r(i,j)} 0.12 b^{kl^+}_{ij} L^f_{ij} + \sum_{c_{kl}\in N_s(i,j)} 0.12 c^{kl^+}_{ij} L^g_{ij} \right\} = 0.0495 < 1,$$

For $1 \leq i, j \leq 2$, take $p = 3$ we have

$$\max_{ij\in\Gamma}\left\{24\left(\frac{2}{3d^-}\right)\left(\frac{4}{3d^-}\right)\left[2\left(\tilde{d}_{ij}^+\right)^3+2\left(\sum_{c_{kl}\in N_r(i,j)}0.12b_{ij}^{kl^+}L_{ij}^f\right)^3\right.\right.$$
$$\left.\left.+\left(1+\frac{2e^{\frac{3}{4}}d^+\beta}{1-\beta'}\right)\left(\sum_{c_{kl}\in N_s(i,j)}0.12c_{ij}^{kl^+}L_{ij}^g\right)^3\right]\right\}=0.1761<1,$$

and, for $p = 2$ we obtain

$$\max_{ij\in\Gamma}\left\{24\left(\frac{2}{d^-}\right)^2\left[2\left(\tilde{d}_{ij}^+\right)^2+2\left(\sum_{c_{kl}\in N_r(i,j)}0.12b_{ij}^{kl^+}L_{ij}^f\right)^2\right.\right.$$
$$\left.\left.+\left(1+\frac{2e^{\frac{1}{2}}d^+\beta}{1-\beta'}\right)\left(\sum_{c_{kl}\in N_s(i,j)}0.12c_{ij}^{kl^+}L_{ij}^g\right)^2\right]\right\}=0.5422<1.$$

For example $i = j = 1,\ k, l = 1, 2 \in N_1(1,1)$,

$$\sum_{c_{kl}\in N_1(1,1)} b_{11}^{kl}(t) f_{11}(z_{kl}(t)) z_{11}(t) = b_{11}^{11}(t) f_{11}(z_{11}(t)) z_{11}(t) + b_{11}^{12}(t) f_{11}(z_{12}(t)) z_{11}(t) + b_{11}^{21}(t) f_{11}(z_{21}(t)) z_{11}(t) + b_{11}^{22}(t) f_{11}(z_{22}(t)) z_{11}(t),$$

$$M_{1ij} = 10, M_{2ij} = 5, M_{3ij} = 6, M_{4ij} = 12 \in \mathbb{R}$$

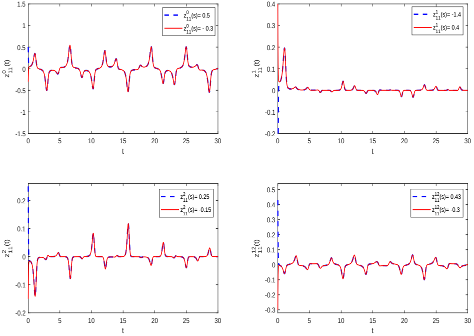

FIGURE – Curves of $z_{11}^{0}(t)$, $z_{11}^{1}(t)$, $z_{11}^{2}(t)$ and $z_{11}^{12}(t)$ of (6) with different initial values.

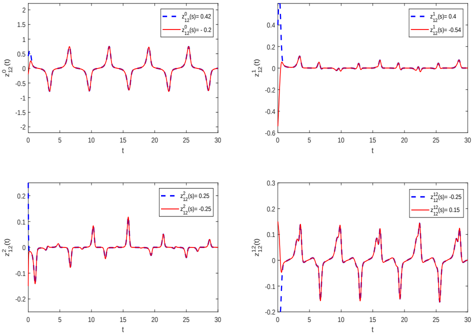

FIGURE – Curves of $z^0_{12}(t)$, $z^1_{12}(t)$, $z^2_{12}(t)$ and $z^{12}_{12}(t)$ of (6) with different initial values.

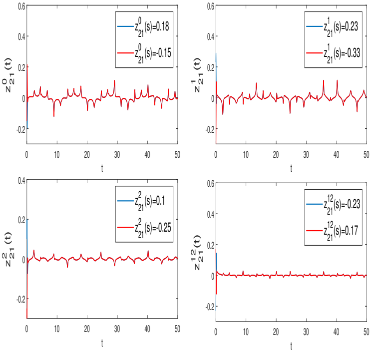

FIGURE – Curves of $z^0_{21}(t)$, $z^1_{21}(t)$, $z^2_{21}(t)$ and $z^{12}_{21}(t)$ of (6) with different initial values.

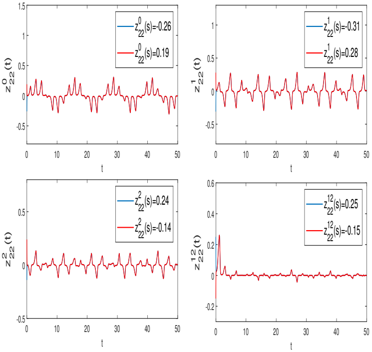

FIGURE – Curves of $z_{22}^{0}(t)$, $z_{22}^{1}(t)$, $z_{22}^{2}(t)$ and $z_{22}^{12}(t)$ of (6) with different initial values.

Plan

Application of synchronization of neural network

*Cryptography Using Artificial Neural Network.

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Cryptography based on delayed chaotic neural networks ☆

Wenwu Yu, Jinde Cao

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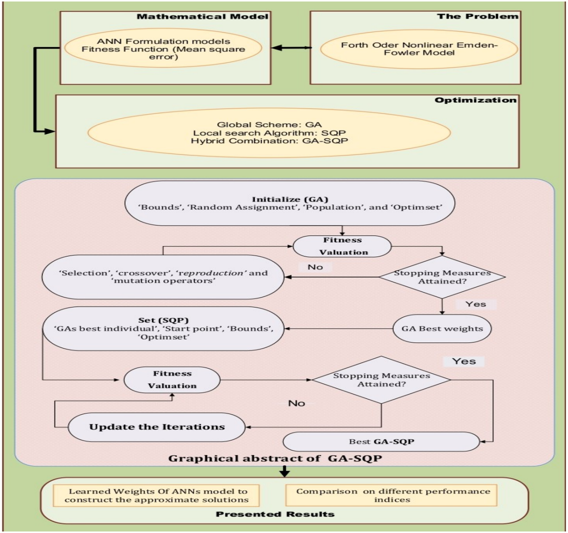

FIGURE – Framework of present methodology for solving the nonlinear fourth

Pseudocode of the optimization scheme of GA-SQP.

GAs process started

Inputs:

The candidate solution with entries equal to unknown parameters in ANN as: $W=[\boldsymbol{\alpha},\boldsymbol{\beta},\boldsymbol{\gamma}]$, where

$\boldsymbol{\alpha}=[\alpha_1,\alpha_2,\alpha_3,\ldots,\alpha_m]$, $\boldsymbol{\beta}=[\beta_1,\beta_2,\beta_3,\ldots,\beta_m]$ and

$\boldsymbol{\gamma}=[\gamma_1,\gamma_2,\gamma_3,\ldots,\gamma_m]$

Population: A set of chromosomes shown as:

$\boldsymbol{P}=[\boldsymbol{W}_1,\boldsymbol{W}_2,\boldsymbol{W}_3,\ldots,\boldsymbol{W}_n]^t$, $\boldsymbol{W}_i=[\boldsymbol{\alpha}_i,\boldsymbol{\beta}_i,\boldsymbol{\gamma}_i]^t$

Output: The decision variable of ANN optimized with GAs as $\boldsymbol{W}_{\mathbf{B.ga}}$

Initialization

Create $\boldsymbol{W}$ with real bounded numbers. Set of $\boldsymbol{W}$ *vector* to make an primary/initial population $\boldsymbol{P}$. Set the values of declarations and Generation of "GA" and "gaoptimset" functions.

Evaluate Fitness

Attained fitness e of $\boldsymbol{P}$ for each weight vector $\boldsymbol{W}$ with the use of equations using equation (7) to (9)

Termination

Terminate the procedure if any of the below condition meets

- Fitness {e $=10^{-20}$}, TolCon = TolFun=10^{-20}
- StallGenLimit=50, PopulationSize=300
- Generations=60, Other values are default

Go to **storage**, when the above criteria meet,

Ranking

Ordered individual $\boldsymbol{W}$ in $\boldsymbol{P}$ for excellence of the fitness

Reproduction

- Selection →"@selectionuniform".
- Crossovers →"@crossoverheuristic".
- Mutations →"@mutationadaptfeasible".
- Elitism →On best individual ranked of population $\boldsymbol{P}$,

Carry on the step of **'fitness evaluation'**

Storage

Save $\boldsymbol{W}_{\mathbf{B.ga}}$, fitness e, time generation and count of

- Since the early 1980s, neural technologies have shown interesting potential for the solving optimization problems. They present in this field, two major advantages : the first lies in the fact that some neural algorithms often solve very well optimization issues ;
- The second comes from the fact that these algorithms are particularly suitable for problems that require extremely short response times. In our work we go focus more specifically on the application of the Hopfield neural network.
- Hopfield neural networks are networks with energy minimization, they are composed of neurons fully connected, they evolve from a state initial.
- They can be used to solve any combinatorial optimization problem, such as that of the traveling salesman, on the condition of deftly modeling the problem.
- The evolution of the network state is based on the minimization of the energy function of the system. So it lends itself very well good at solving optimization problems.

- For the optimization, we use the network of the following way : from an initial state, we leave the network evolve freely up to an attractor, which is typically, for optimization problems, a state stable independent of time (a fixed point of the dynamics). We then says that the network has converged : convergence is reached when the outputs of the neurons no longer evolve.
- The dynamics of the network are generally asynchronous : between two instants of time, a single neuron, chosen randomly, is update ; in other words, its potential is calculated, and its output re-evaluated accordingly.
- When these networks are used to solve problems optimization, the weights of the connections are determined analytically from the formulation of the problem ; in general, this is done directly from the function energy associated with the problem. In addition, the outputs of neurons, in the attractor towards which the network converges, encode a solution to the optimization problem.

Codage (Traveling Salesman problem)

	Etape 1	Etape 2	Etape 3	Etape 4	Etape 5	Etape 6
Ville A	1	0	0	0	0	1
Ville B	0	1	0	0	0	0
Ville C	0	0	1	0	0	0
Ville D	0	0	0	1	0	0
Ville E	0	0	0	0	1	0

Let $V(x,i)$ is the output of neuron in the position (x,i) of our matrice.

$$E_1 = \sum_x \sum_i \sum_{j \neq i} V(x,i) \cdot V(x,j)$$

$$E_2 = \sum_i \sum_x \sum_{y \neq x} V(x,i) \cdot V(y,i)$$

$$E_3 = (\sum_{x,i} V(x,i) - N)^2$$

$$E_4 = \sum_x \sum_{y \neq x} \sum_i [d_{xy} \cdot V(x,i) \cdot (V(y,i+1) + V(y,i-1))].$$

$$E = -a \cdot E_1 - b \times E_2 - c \cdot E_3 - d \cdot E_4,$$

with $(a,b,c,d) \in \mathbb{R}^4$.
Identification of the energie E with the Hopfield energie.

Thank you for your attention